Errors in S_{11} Measurements due to the Residual Standing-Wave Ratio of the Measuring Equipment

ROBERT V. GARVER, SENIOR MEMBER, IEEE, DIETRICH E. BERGFRIED, MEMBER, IEEE, SAMUEL J. RAFF, SENIOR MEMBER, IEEE, AND BRUNO O. WEINSCHEL, FELLOW, IEEE

***Abstract*—Errors in measurement of S_{11} due to the residual SWR of the slotted line or directional coupler are calculated using admittance considerations. This more complete treatment produces, for some practical conditions, major differences from prior references for phase errors.**

I. Introduction

ALTHOUGH slotted lines have been used for measuring impedance for over three decades and directional couplers have been used at least for one decade, the phase errors due to the residual SWR and the finite directivity of these devices have never been correctly calculated. The mathematics of the phase error, for the general case, have so far proven to be intractable. Results are presented here of a computer program in which these errors were calculated for a wide range of parameters using numerical methods. The phase errors are compared to the relationships that have been used incorrectly by others. The phase errors are shown to be very closely described by the equation

$$\Delta\phi = \left(1 + 3\,|\Gamma_x| - \frac{1}{4}\sin\pi\,|\Gamma_x|\right)\sin^{-1}\left|\frac{\Gamma_m}{\Gamma_x}\right| \quad (1)$$

in which Γ_x is the reflection coefficient of the unknown and Γ_m is the reflection coefficient of the intervening mismatch when a perfect load is placed behind it (residual SWR).

The errors discussed in this paper apply to the input reflection coefficient of a network that is measured with all other ports terminated into matched loads. The input reflection coefficient will be called Γ_x when one-ports are discussed and S_{11} when two-ports are considered. The theory is centered around one-ports for simplicity. The results however are equally valid for the general multiport terminated into matched loads.

An equivalent description of the reflection is given by the term SWR.

The magnitude of the error due to the residual SWR of the measuring equipment using this notation is given most simply by the equation

$$\mathrm{SWR}_R = (\mathrm{SWR}_x)(\mathrm{SWR}_m)^{\pm 1} \quad (2)$$

which is in agreement with the expression given by Altman [1] as discussed later.

The phase errors given by the manufacturers of precision phase measurement equipment are given [2] in Table I.

TABLE I

Error	Manufacturer
$\pm\left[A+\tan^{-1}\frac{B}{\lvert\Gamma_x\rvert}+C\lvert\Gamma_x\rvert\right]$	
$\pm\left[2°+\tan^{-1}\frac{0.01}{\lvert\Gamma_x\rvert}+3°\lvert\Gamma_x\rvert\right]$	Rantec
$\pm\left[2°+\tan^{-1}\frac{0.01}{\lvert\Gamma_x\rvert}+3.5°\lvert\Gamma_x\rvert\right]$	Wiltron
$\pm\left[0.5°+\tan^{-1}\frac{0.003}{\lvert\Gamma_x\rvert}+4\tan^{-1}(0.015\lvert\Gamma_x\rvert)\right]$	Hewlett-Packard (computer-corrected)
$\pm\left[0.25°+\tan^{-1}\frac{0.0015}{\lvert\Gamma_x\rvert}+4\tan^{-1}(0.005\lvert\Gamma_x\rvert)\right]$	Hewlett-Packard (computer-corrected) with phase lock)

Error A is contributed mostly by the low frequency information processing circuits. The factor B is contributed by the finite directivity of the directional coupler used in picking up the reflected wave. Directivity η is given by

$$\eta = 20 \log B.$$

Thus $B = 0.01$ indicates 40-dB directivity. The factor C is contributed by residual output SWR. This factor was originally taken as $\tan^{-1}|\Gamma_m| \approx 60°|\Gamma_m|$. A residual output SWR of 1.1 would give $|\Gamma_m| = 0.05$ which would give $C = 3°$. Lately, Hewlett-Packard has begun to use $4\tan^{-1}|\Gamma_m|$.

The equations of Table I for phase errors are in error in two respects: first, the equations for residual SWR are not correct, and second, directional coupler directivity errors must be combined with residual SWR effects before (1) is applied. The manufacturers have been careful in not publishing explanations of the errors they have specified. The equations they give may fit their observed errors but the error mechanism is not correctly associated with the error. In fact an output SWR of 1.2 ($|\Gamma_m| = 0.091$) is common in specifications which for $|\Gamma_x| = 1$ would give a maximum error of $\pm 20.8°$ using (1), but according to Table I, the maximum error

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D. E. Bergfried and B. O. Weinschel are with the Weinschel Engineering Company, Gaithersburg, Md. 20760.
R. V. Garver and S. J. Raff are Private Consultants to the Weinschel Engineering Company, Gaithersburg, Md. 20760.

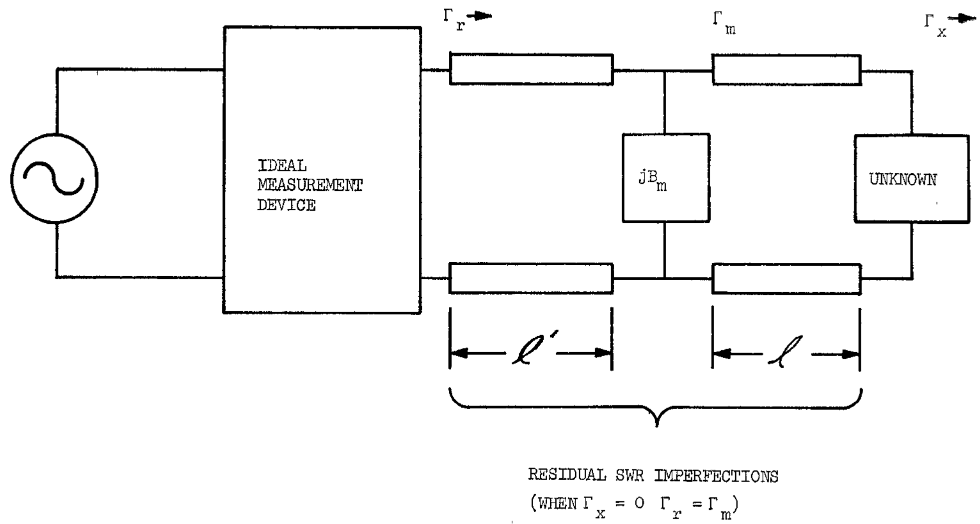

Fig. 1. Microwave circuit for S_{11} measurements.

for the noncomputer-corrected measurement would be about $\pm 6°$.

II. Theory

The residual SWR may be the summation of smaller SWRs contributed by connector irregularities or transmission line imperfections. Assuming all of these discontinuities to be lossless then the measuring system can be represented by the general transmission line circuit shown in Fig. 1. The characteristic admittances of transmission line l' and l are the same as the output admittance of the ideal measuring device. B_m and the unknown are also normalized to this same characteristic admittance. Three degrees of freedom are given by l', l, and B_m. The circuit is general because it can transform any point on the Smith chart into any other point. The residual mismatch Γ_m is determined by connecting a known admittance on the unknown port having $\Gamma_x = 0$. Under this condition the resulting reflection coefficient is $\Gamma_r = \Gamma_m$. The electrical plane of the discontinuity B_m is not necessarily in the plane of the connector or in the plane of the unknown, but it may be in any plane as required to represent the intervening mismatch as seen by the ideal measuring device.

The value of B_m may be calculated by setting $\Gamma_x = 0$. Then the normalized admittance of line length l and the "unknown" Γ_x is independent of the length l. Length l' is adjusted until the admittance as measured in the plane of B_m lies on the $G=1$ curve on the Smith chart. This rotation may be done until the $G=1$ circle is intercepted with either B_m positive or B_m negative. The positive sign for B_m is arbitrarily taken giving an equivalent shunt capacitor for B_m. Negative B_m could be taken just as easily giving equal phase errors later of the opposite polarity. The reflection coefficient of the intervening mismatch Γ_m in the plane of B_m is given by

$$\Gamma_m = \frac{1 - Y_m}{1 + Y_m} \tag{3}$$

in which

$$Y_m = 1 + jB_m. \tag{4}$$

Solving for B_m gives

$$B_m = \frac{2\,|\Gamma_m|}{\sqrt{1 - |\Gamma_m|}} = \rho_m^{1/2} - \rho_m^{-1/2} \tag{5}$$

in which ρ_m is the residual SWR_m of the measuring equipment

$$\rho_m = \frac{1 + |\Gamma_m|}{1 - |\Gamma_m|}. \tag{6}$$

The manner in which the discontinuity causes phase error is shown in Fig. 2. An $SWR_m = 1.5$ is used in conjunction with a short circuit which has a normalized admittance of $Y_x = \infty + j\infty$. This mismatch gives an equivalent susceptance B_m of $B_m = 0.409$. The Smith chart of Fig. 2 shows admittance adding in the plane of the equivalent discontinuity B_m for various values of l/λ, the electrical distance of the short circuit to the equivalent discontinuity. The greatest phase shift occurs when the discontinuity susceptance takes the total admittance from $0 - jB_m/2$ to $0 + jB_m/2$. Here the phase shift is 46°. If the opposite sign had been selected for B_m the phase error would be in the opposite direction. Note that for one assumed value of B_m the phase error varies from 0 to a maximum depending on l/λ. Two measurements made through the same discontinuity (as with the initial and final settings of the standard phase shifter) will not have additive errors. The worst case would be where the one phase setting gives the maximum error and the other one gives none.

It has been assumed that the phase error is caused by reactive discontinuities. This is most often the case. It is also the worst case. Calculations of S_{11} errors from resistive discontinuities give smaller errors.

The admittance adding process for a reflection coefficient of $|\Gamma_x| = 0.5$ and $SWR_m = 1.5$ is shown in Fig. 3. In order to simplify the calculation, the observer moves

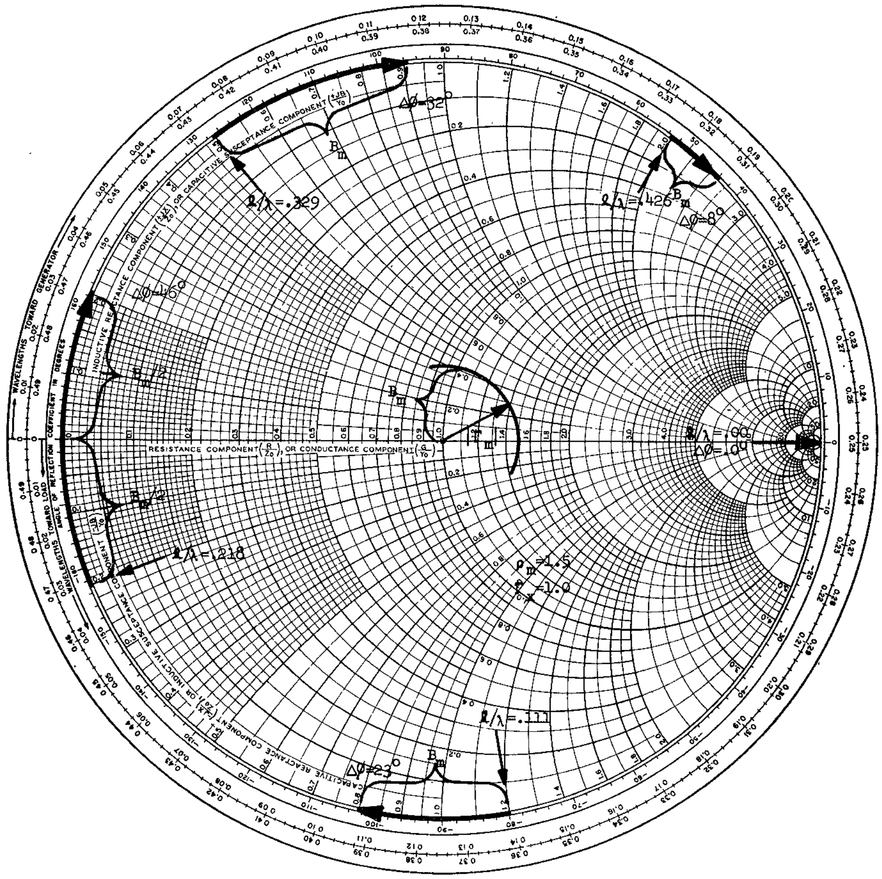

Fig. 2. Phase errors possible with $\rho_m = 1.5$ and $|\Gamma_x| = 1.0$.

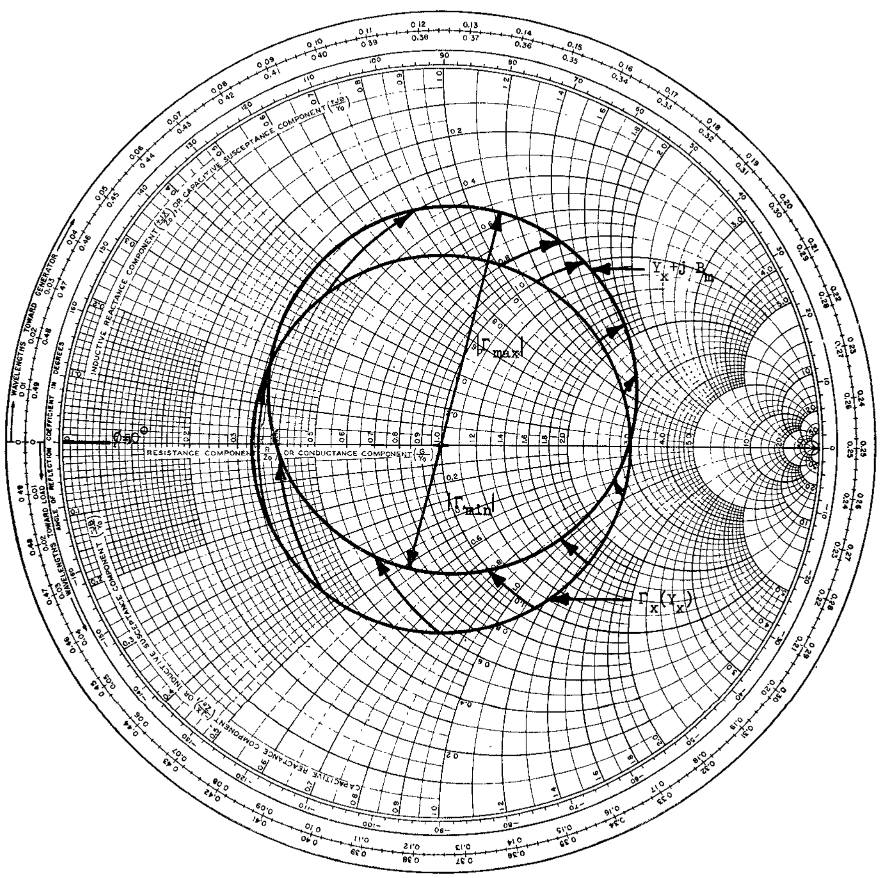

Fig. 3. Range of possible new admittance points for $|\Gamma_x| = 0.5$ and $\rho_m = 1.5$ assuming B_m positive. Admittances are in the plane of Γ_m.

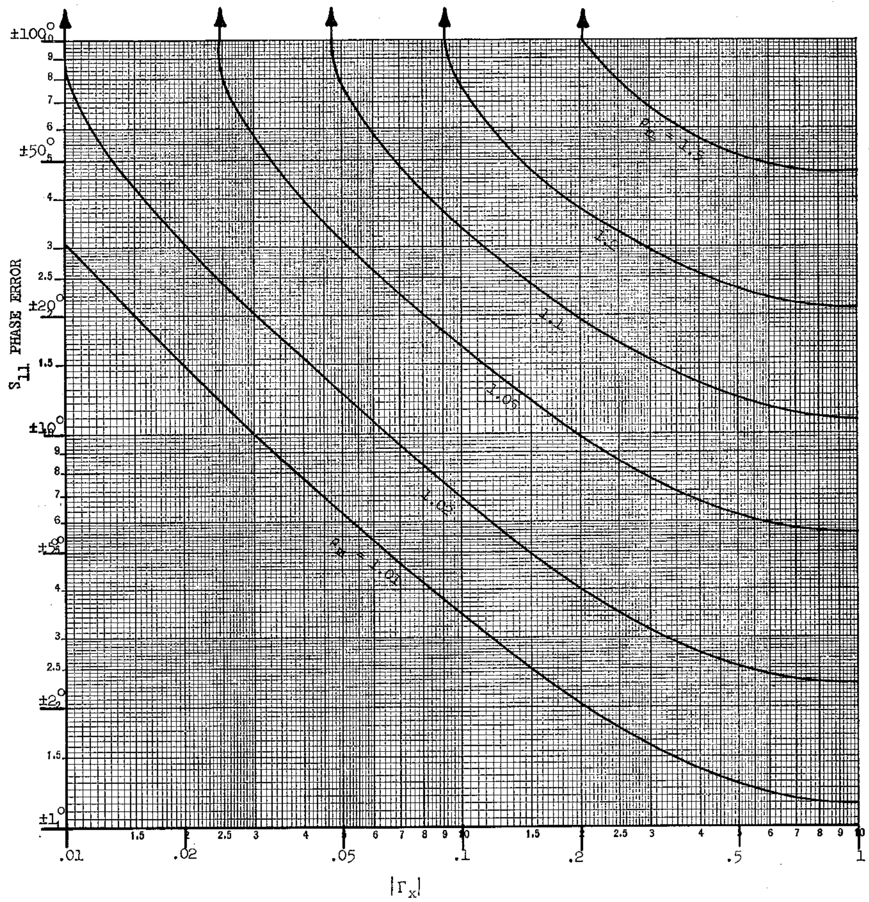

Fig. 4. Computed maximum phase errors in S_{11} for various values of $|\Gamma_x|$ and ρ_m.

from the unknown toward the generator until the normalized admittance of the unknown is real and greater than one ($Y_x = 3.0 + j0$ for Fig. 3). When l/λ is measured from this position the process used in Fig. 3 is similar to that used in Fig. 2. As l/λ is increased the circle labeled Y_x becomes the admittances of the unknown. When the susceptance of the discontinuity is added the circle $Y_x + jB_m$ is generated. The maximum phase shift occurs again nearest $0+j0$ and is almost centered around the real axis. Fig. 3 also shows what happens to the magnitude of reflection coefficient. The resulting SWR, SWR_R is given by (2), the + sign giving the maximum and the − sign, the minimum. Equation (2) is in agreement with Altman [1] who gives

$$|\Gamma_R| = \frac{|\Gamma_m| \pm |\Gamma_x|}{1 \pm |\Gamma_m||\Gamma_x|}. \tag{7}$$

But the phase shift is not in agreement with any of the known expressions.

III. Numerical Method

Many attempts to derive an analytical expression for the phase error have led to intractable arrays of algebra. Therefore, a numerical method was set up on the computer doing the same operations as were performed on Fig. 3. Maximum phase error occurs for the phase of the unknown reflection coefficient ϕ_x when $0° < \phi_x < 90°$. The computation is performed as follows:

1) with $\phi_{x1} = 0°$ then Y_{x1} is calculated from $|\Gamma_x| \epsilon^{j\phi_{x1}}$;
2) B_m is added giving a new admittance Y_x' and Γ_x';
3) ϕ_{x1}' is calculated from Γ_x';
4) $\Delta\phi_1 = \phi_{x1}' - \phi_{x1}$;
5) ϕ_{x1} is incremented by $1° (\phi_{x2} = \phi_{x1} + 1°)$;
6) Y_{x2} is calculated; repeat steps 2), 3), and 4) getting $\Delta\phi_2$;
7) $\Delta\phi_2$ is compared to $\Delta\phi_1$; if $\Delta\phi_{i+1} < \Delta\phi_i$ then the calculation is stopped and $\Delta\phi_i$ is displayed as the maximum error for the given $|\Gamma_x|$ and B_m.

If $\Delta\phi_{i+1} \geq \Delta\phi_i$ then steps 5) through 7) are repeated.

The program and results using a Hewlett-Packard model 9100 calculator are shown in the Appendix.

Using this method the phase error curves were calculated as shown in Fig. 4. An equation has been found which fits these curves within a few percent which was given as (1).

IV. Analytical Solutions of the S_{11} Measurement Errors

A. Exact Calculation for $|\Gamma_x| = 1$

For $|\Gamma_x| = 1$ the susceptance B_m causes the largest phase error when it takes the admittance in the plane of the mismatch from $0-jB_m/2$ to $0+jB_m/2$. A susceptance $B_m/2$ can be generated by displacing an open circuit a distance $\Delta l/\lambda$ satisfying

$$\frac{B_m}{2} = \tan 2\pi \frac{\Delta l}{\lambda} \cdot \quad (8)$$

In order to obtain the phase error $\Delta\phi$ the term $\Delta l/\lambda$ must be multiplied by a factor of two because the reflected wave makes a two-way pass to the new open circuit and by another factor of two because only half of B_m was used for the function.

Thus

$$\Delta\phi = 2 \times 2 \times 2\pi \frac{\Delta l}{\lambda} \quad (9)$$

which when substituted into (8) gives

$$\frac{B_m}{2} = \tan\left(\frac{\Delta\phi}{4}\right) \quad (10)$$

$$\Delta\phi = 2\tan^{-1}\left[\frac{2B}{1-G^2-B^2}\right] = \tan^{-1}\left[\frac{2(\rho_m^{1/2}-\rho_m^{-1/2})}{2+(\rho_x+\rho_x^{-1})\sqrt{(\rho_x^2+\rho_x^{-2})}-(\rho_m+\rho_m^{-1})-(\rho_x^2+\rho_x^{-2})}\right]. \quad (21)$$

recalling (5) produces

$$\frac{B_m}{2} = \frac{|\Gamma_m|}{\sqrt{1-|\Gamma_m|^2}} = \tan\left(\frac{\Delta\phi}{4}\right) \quad (11)$$

and recalling that

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} \quad (12)$$

gives with (11)

$$\Delta\phi = 4\sin^{-1}|\Gamma_m|. \quad (13)$$

Equation (13) is exact for $|\Gamma_x| = 1$ and, consequently, agrees exactly with the computed results and (1).

B. Approximation for $|\Gamma_x| > 4|\Gamma_m|$

For $|\Gamma_x| \neq 1$ the assumption is made that the maximum error occurs when the center of the arc of B_m is on the $B=0$ axis of the Smith chart with $G<1$ (G is normalized). This assumption very accurately represents S_{11} measurement for $|\Gamma_x| > 4|\Gamma_m|$. Given $|\Gamma_x|$ find G satisfying $-B_m/2$ and $|\Gamma_x|$ then

$$\Delta\phi = 2\tan^{-1}\left[\frac{\text{Im}(\Gamma_x)}{\text{Re}(\Gamma_x)}\right]. \quad (14)$$

Define

$$-B_m/2 = -B \quad (15)$$

then

$$\Gamma_x = \frac{1-G+jB}{1+G-jB} \quad (16)$$

and solving for G gives

$$G = \left(\frac{1+|\Gamma_x|^2}{1-|\Gamma_x|^2}\right) \pm \sqrt{\left(\frac{1+|\Gamma_x|^2}{1-|\Gamma_x|^2}\right)^2 - (1+B^2)}. \quad (17)$$

The minus sign is taken for $G<1$ and (17) can be written

$$G = \tfrac{1}{2}(\rho_x + \rho_x^{-1}) - \sqrt{\tfrac{1}{4}(\rho_x^2 + \rho_x^{-2} + 2) - (1+B^2)}. \quad (18)$$

Equation (16) is written in normalized form as

$$\Gamma_x = \frac{(1-G)+jB}{(1+G)-jB}\cdot\frac{(1+G)+jB}{(1+G)+jB} = \frac{(1-G^2-B^2)+j2B}{(1+G)^2+B^2} \quad (19)$$

which produces

$$\frac{\text{Im}(\Gamma_x)}{\text{Re}(\Gamma_x)} = \frac{2B}{1-G^2-B^2} \quad (20)$$

Equation (21) is indistinguishable from computer curves of Fig. 4 for $|\Gamma_x| > 4|\Gamma_m|$ and only 1–2 percent low at $|\Gamma_x| = 2|\Gamma_m|$. Although this expression is analytic and accurate for most practical applications, (1) is more convenient to use.

C. Approximation Based on S-Parameter Equation

The resultant reflection coefficient Γ_R as observed looking at an unknown Γ_x through a scattering matrix S_{mn} is given by Schafer [3]

$$\Gamma_R = S_{11} + \frac{S_{12}S_{21}\Gamma_x}{1-S_{22}\Gamma_x} \cdot \quad (22)$$

The equivalent mismatch due to a residual SWR is represented by a single reactive element which is reciprocal and lossless. The scattering matrix for such an element has the following identities: $S_{11}=S_{22}$, $S_{21}=S_{12}$, and $S_{21}=1+S_{11}$. Intervening mismatch Γ_m corresponds to S_{11} thus (22) reduces to

$$\Gamma_R = \Gamma_m + \frac{(1+\Gamma_m)^2\Gamma_x}{1-\Gamma_m\Gamma_x} = \frac{\Gamma_x+\Gamma_m+2\Gamma_x\Gamma_m}{1-\Gamma_x\Gamma_m} \cdot \quad (23)$$

The error in phase is derived by taking the quotient Γ_R/Γ_x since this will give the difference in phase between the two vectors.

$$\frac{\Gamma_R}{\Gamma_x} = \frac{1+\Gamma_m/\Gamma_x+2\Gamma_m}{1-\Gamma_m\Gamma_x} \cdot \quad (24)$$

The vector diagram of Fig. 5 shows how these vectors could add up to give the greatest phase error. The maximum phase error would be given by

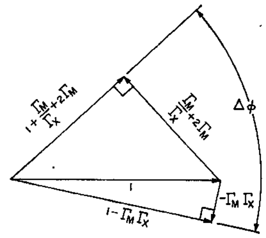

Fig. 5. Vector addition of (24) for worst case phase error.

$$\Delta\phi = \sin^{-1} |\Gamma_m \Gamma_x| + \sin^{-1} \left| \frac{\Gamma_m}{\Gamma_x} + 2\Gamma_m \right| \quad (25)$$

if the vectors could be arranged as shown. Worst case phases cannot be used because the vector products interact with each other. The vectors have only two degrees of freedom to begin with: the phase of Γ_m and the phase of Γ_x. But one degree of freedom has already been lost with the assumption that the mismatching susceptance is reactive and subsequently that the scattering matrix of the intervening mixmatch is lossless. Three vectors cannot be maximized easily with one degree of freedom. Different combinations of the terms were tried. One combination was found to fit the computer data with half the deviation of (1) for $|\Gamma_x| \approx |\Gamma_m|$ but at the cost of a slight deviation at $|\Gamma_x| = 1$ for large $|\Gamma_m|$.

$$\Delta\phi = \sin^{-1} |\Gamma_m \Gamma_x| + \sin^{-1} |\Gamma_m / \Gamma_x| + \sin^{-1} |2\Gamma_m| . \quad (26)$$

V. Discussion of Errors

The subject of uncorrected errors in S_{11} phase contributed by intervening mismatches between the measuring device and the device being measured has not been directly addressed to date in the literature. Most of the interest has been in errors in measuring a reciprocal phase shifter by the method of adjusting a calibrated short circuit behind it and keeping a fixed minimum on a slotted line [3], [4]. Errors are introduced by the SWR of the phase shifter. Using a directional coupler the sliding short circuit can be used as a standard phase shifter [4], [5]. Errors introduced by the finite directivity of the directional coupler were carefully analyzed [6], [7]. This same directional coupler error analysis may be adapted to S_{11} measurements in which a microwave bridge for measuring S_{21} is converted to measure S_{11} by using the directional coupler to convert the reflected wave to a through wave. But all of the error analyses were concerned with phase shift—the difference between an initial and a final setting of the phase shifter. The error in S_{11} phase is the difference between the measured value and the correct value.

A. Vector Crank Model

Magid [4] and Schafer [3] have considered the errors in measuring a reciprocal phase shifter by adjusting a calibrated short behind it to keep a fixed minimum on a slotted line in front of the phase shifter. Magid [4] referencing others considered the error to be represented by a vector crank model addition of reflection coefficients. The sliding short giving a unity reflection coefficient and the SWR of the phase shifter giving a small error-causing reflection coefficient Γ_m. The error is given by

$$\Delta\phi = \sin^{-1} |\Gamma_m| . \quad (27)$$

The same vector crank model for a nonunity load $|\Gamma_x|$ would give

$$\Delta\phi = \sin^{-1} |\Gamma_m / \Gamma_x| . \quad (28)$$

Comparison with (1) will show these expressions are too small by a factor of 4. It is interesting that Magid even conducted an experiment to verify the relationship. He created a 1.35 SWR with a tuner between his ideal measuring device and sliding short. Measuring the deviation from linearity on the probe position as the short circuit is moved he observed 18.2° error peak to peak which he concluded was in good agreement with ±8.55° predicted by (27). What he did not observe and would have had he used a capacitive probe instead of an E/H tuner is that his error in fact goes from 0° to 18.2° as he calculated it or 0° to 36.4° using $4\pi\Delta l/\lambda$ (which gives the angle of S_{11}) instead of $2\pi\Delta/\lambda$ which gives the one-way phase shift of the reciprocal phase shifter normally under test. It is shown in Fig. 6 that (28) is in marked disagreement with the computed errors.

The vector crank model as applied to the errors in S_{11} phase measurements has been shown to be in error by a factor of four. The vector crank model is used quite extensively for S_{21} measurement errors. Is it incorrect for S_{21} measurements also? No. The mechanism for power splitting and power recombining in S_{21} measurements is resistive. The vector crank model is correct for adding powers at resistors. But when waves are combined at reactances as in S_{11} measurements then admittance adding processes or careful consideration of scattering matrices must be used.

Suppose a short is connected to a network analyzer (using directional couplers for S_{11} measurements) and the $|\Gamma_R| = 1$ circle is observed. Using reactance agruments for phase errors the circle would be perfect having no fluctuations about unity. But fluctuations about unity occur. Would this not support the voltage vector addition (vector crank) model as correct for calculating errors? The errors in magnitude for high VSWR impedances are not caused by the output discontinuities of the directional coupler but instead they are caused by the mismatch to the directional coupler seen looking back into the generator.

A method in general use for correcting for the residual mismatch of a directional coupler in a network analyzer

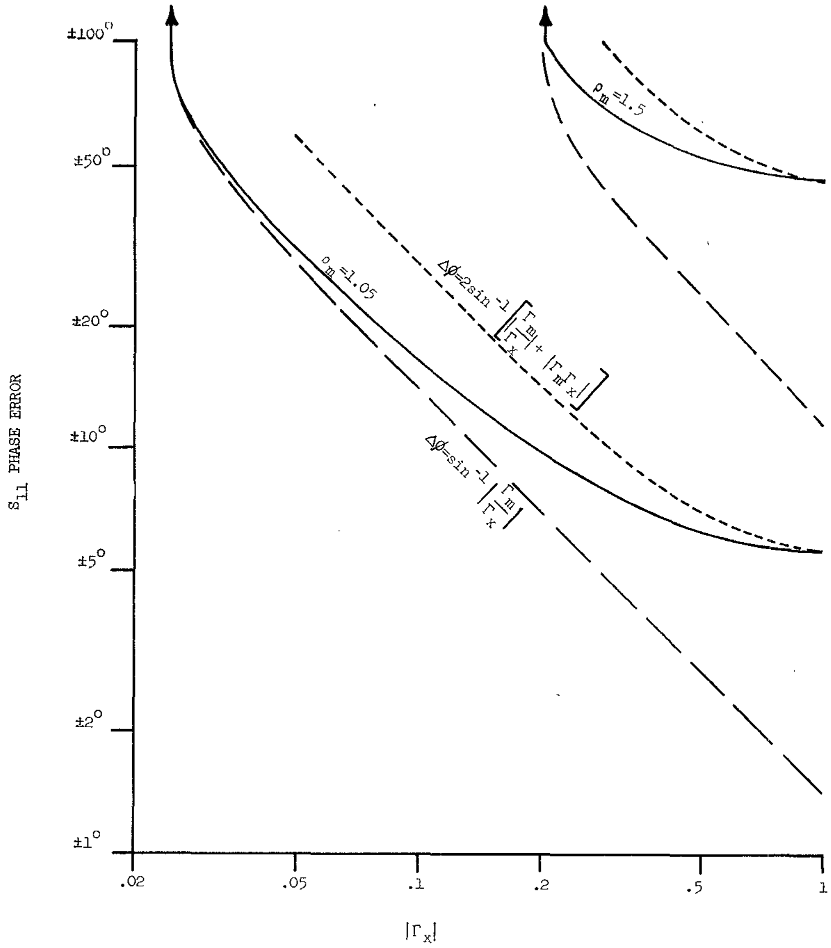

Fig. 6. Comparison of various calculations of phase errors in S_{11} for $\rho_m = 1.05$ and 1.5.

at a single frequency is to use a sliding load to generate an admittance circle and then center the circle on the Smith chart with the network analyzer CRT centering adjustments. If the voltage vector addition model of errors could be correctly applied here then this method of correction would be valid for all values of Γ_x but since it cannot, the method can give quite large phase errors for small values of $|\Gamma_x|$.

B. Phase Shifter Measurement

Schafer [3] carried the error analysis of the fixed slotted line probe method farther by starting with (22). When the condition is satisfied for keeping the minimum on the slotted line the same for initial and final settings of the phase shifter and sliding short then Schafer obtains

$$\Delta\phi' = \sin^{-1}\left[\frac{1}{2}\,\frac{|{}^fS_{11}| + |{}^fS_{21}{}^2\,{}^fS_{22}\,{}^f\Gamma_x{}^2|}{|{}^fS_{21}{}^2\,{}^f\Gamma_x|} + \frac{1}{2}\,\frac{|{}^iS_{11}| + |{}^iS_{21}{}^2\,{}^iS_{22}\,{}^i\Gamma_x{}^2|}{|{}^iS_{21}{}^2\,{}^i\Gamma_x|}\right] \tag{29}$$

in which superscripts f and i represent final and initial phase shifter settings and $\Delta\phi'$ relates to the one-way phase error of the reciprocal phase shifter ($\Delta\phi = 2\Delta\phi'$). A fixed scattering matrix as considered here will be the same in initial and final settings. Using Schafer's substitution that $|S_{21}| \approx 1$ and Γ_m as defined here $\Gamma_m = S_{11}$ of the intervening scattering matrix then (29) reduces to

$$\Delta\phi = 2\sin^{-1}(|\Gamma_m/\Gamma_x| + |\Gamma_m\Gamma_x|). \tag{30}$$

Again this error would be the maximum error between two phase settings. Equation (30) is in substantial agreement with (1) only when $\Gamma_x = 1$, as shown in Fig. 6.

Schafer's analysis [3] of maximum error was based on the assumption that the phase error could be opposite for the two phase states. Since his scattering matrix (phase shifter) is changing between states such error is possible. But his expression of error is not large enough because he made the assumption that $|S_{21}| = 1$ contributes magnitude but not phase to the vector diagram while in fact it contributes the same amount to phase error as the other terms, thus doubling his expression. His assumption that S_{11} and S_{22} do not change between phase states should have halved his error because he assumed that in the worst case his errors in each state added while in fact when S_{11} and S_{22} do not change because they have the same sign and at worst one is a maximum and the other is zero. If S_{11} and S_{22} change between phase states then Schafer's equation is too low by a factor of two.

C. Standard Phase Shifter

Beatty and Kerns [5], and Magid [4] simultaneously reported the method of using a sliding short circuit on a directional coupler as a standard phase shifter. This standard phase shifter has been subsequently refined by Schafer and Beatty [6] and by Ellerbruch [7]. Shafer and Beatty [8] derived a maximum error limit $\Delta\phi$ given by

$$\Delta\phi = 2\left|\frac{S_{31}}{S_{32}S_{21}}\right|\left|\sin\left(\frac{\varphi_L}{2}\right)\right| \quad \text{rad} \tag{31}$$

in which φ_L is the difference between the initial and final phases of the power reflected from the sliding short circuit, S_{31} is input-to-output port (undesired) coupling, and S_{32} is the coupling of the reflected wave. $|S_{31}/S_{21}S_{32}|$ is normally called directivity. 20-dB directivity gives $|S_{31}/S_{21}S_{32}| = 0.1$. When $\varphi_L = 180°$ then the 20-dB directivity would give ± 0.2-rad error ($\pm 11.4°$). Magid [4] showed that this directivity error could be canceled with a tuner on the no. 2 port of the directional coupler. To cancel a wave having $|S_{31}/S_{21}S_{32}| = 0.1$ the tuner must have a reflection coefficient of 0.1 ($\Gamma_m = 0.1$). The sliding short is equivalent to $\Gamma_x = 1.0$. Equation (1) indicates the phase error for one setting should be $\pm 22.8°$. 180° away from this setting the error would be close to $\pm 0°$ so (31) is too small by a factor of two.

A close look at the derivation of (31) by Schafer and Beatty reveals a possible source of error. The assumption that $|S_{21}| \approx 1$ leads to omitting the phase contribution of S_{21} which above lead to a factor of two error. Since a tuner (shunt susceptance) can completely eliminate the undesired coupling in a directional coupler then the mechanism for phase error must be the same (addition of undesired susceptance).

VI. Conclusions

The errors in S_{11} measurements due to the residual SWR of the measuring equipment have been calculated using admittance adding considerations. The errors in magnitude agree with well-accepted expressions used in the past as given by (2). The phase errors however have never been correctly derived and are given graphically in Fig. 4, or approximately by (1) and (26). The errors for directional couplers must be converted to residual SWR errors and combined with other residual SWRs using (2) in order to be correct.

Further work is needed to find satisfactory analytical expressions for phase errors. The errors associated with standard phase shifters (sliding short or a directional coupler) are presently too small by a factor of two. Greater care must be exercised in obtaining low VSWR outputs to obtain accurate S_{11} phase information. Manufacturers presently specify output VSWR that will give much higher S_{11} phase error than they say they can obtain. The method of moving an S_{11} circle for a sliding load to the middle of the CRT on a network analyzer does not give all of the correction that users might think. Phase errors continue to be quite large.

Appendix

S_{11} Phase-Error Program for HP 9100 Calculator

	0	1	2	3	4	5	6	7
0	√	d	c	Rect	e^x	c	y	9
1	↑	x→	↑	xy	Rect	xy	[illegible]	end
2	1	c	a	↑	xy	−	GT	
3	±	↑	Rect	b	×	d	1	
4	xy	a	↑	+	↑	Pause	4	
5	÷	±	1	↓	2	if x>y	↑	
6	+	Rect	+	xy	R↑	6	c	
7	y→	↑	↓	↑	−	5	↑	
8	b	1	Pol	1	↓	y→	1	
9	Stop	+	ln x	+	Pol	d	−	
a	x→	↓	Σ−	↓	ln x	c	a	
b	a	Pol	RCL	Pol	Σ+	↑	R↓	
c	0	ln x	Σ−	ln x	RCL	1	GT	
d	x→	Σ+	e^x	Σ−	Σ−	+	0	

Angle set to degrees

a: $|\Gamma_x|$

b: B_m

c: ϕ

d: $\phi - \phi'$

Enter ρ_m
Press Cont
Enter $|\Gamma_x|$
Press Cont
At pause y = new $\Delta\phi$
At stop z = $|\Gamma_x|$
y = $\Delta\phi$
= ϕ

For new $|\Gamma_x|$ enter $|\Gamma_x|$ and press cont.

For new ρ_m enter ρ_m and press end then cont.

To speed calculation increment is instruction 5c

Change 11 and 12 to continue and enter beginning ϕ in c.

Calculated S_{11} Phase Errors

ρ_m=1.5

$\lvert\Gamma_x\rvert$	$\Delta\phi$ (ϕ)
.999	46.15°(23°)
.5	52.16°(32°)
.4	57.23°
.3	67.54°(48°)
.25	77.95°(60°)
.22	89.48°(78°)
.21	96.10°(83°)
.2	112.83°(101°)

ρ_m =1.2

$\lvert\Gamma_x\rvert$	$\Delta\phi$ (ϕ)
~~.999~~	~~20.86°~~
.5	23.48°
.26	32.18°(24°)
.2	38.40°(30°)
.13	55.28°(48°)
.115	63.04°(59°)
.1	75.98°(72°)
.095	83.70°(78°)

ρ_m =1.1

$\lvert\Gamma_x\rvert$	$\Delta\phi$ (ϕ)
~~.999~~	~~10.92°~~
.5	12.28°
.2	19.68°
.1	34.13°(30°)
.08	42.11°(42°)
.06	58.07°(54°)
.05	77.55°(72°)
.048	88.13°(84°)

ρ_m =1.05

$\lvert\Gamma_x\rvert$	$\Delta\phi$ (ϕ)
.999	5.59°
.5	6.29°
.2	10.07°
.1	16.95°
.07	23.26°(24°)
.05	32.05°(30°)
.038	42.76°(42°)
.03	57.17°(54°)
.025	80.10°(78°)

ρ_m =1.02

$\lvert\Gamma_x\rvert$	$\Delta\phi$ (ϕ)
.999	2.27°
.5	2.55°
.2	4.08°
.1	6.87°
.05	12.53°
.03	20.41°(18°)
.02	30.82°(30°)
.014	46.10°(48°)
.012	56.68°(54°)
.011	65.31°(65°)
.01	82.90°(84°)

ρ_m =1.01

$\lvert\Gamma_x\rvert$	$\Delta\phi$ (ϕ)
.999	1.14°
.5	1.28°
.2	2.05°
.1	3.45°
.05	6.29°
.02	14.96°(12°)
.01	30.41°(30°)
.008	38.98°(36°)
.006	56.52°(54°)
.0054	67.69°(67°)
.005	84.83°(84°)

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Application of Computer-Controlled Spectrum Surveillance Systems to Crime Countermeasures

H. DEAN McKAY, MEMBER, IEEE

Abstract—**A computer-controlled spectrum surveillance system, and application of such a system to crime countermeasures, is described. The system covers the frequency range of 5 kHz to 12 GHz. All functions are controlled via mini-computer, with output directly compatible with batch processing computers.**

INTRODUCTION

THE NEED to utilize the electromagnetic spectrum as a crime countermeasures tool has been recognized for a number of years. All organizations involved in fighting crime use communications today in their operations. Such tools as high-speed data links, radar, and computers have also been integrated into the crime countermeasures field. It has become extremely important that those involved in combatting crime have complete control and management of the electromagnetic spectrum in which they intend their operations.

Heretofore, crude attempts have been made to provide limited-range frequency surveillance which could be utilized in the frequency-management task. However, such systems required manual operation, interpretation by highly trained operators, and, in general, a large quantity of equipment which was not easily transported and not at all compatible with high-speed data processing.

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The author is with Fairchild/Electro-Metrics Corporation, Panorama City, Calif. 91402.

In this discussion an attempt is made to describe equipment and systems which can be used in this function. They are, in general, commercially available and can be operated by variably untrained technical personnel.

Some examples are given of the applications these systems can be utilized for. No attempt will be made to get into the detail system requirements. However, the basic design tradeoffs and interface requirements are discussed.

SYSTEM DESCRIPTION

The spectrum surveillance system discussed in this technical paper differs from other swept-tuned receivers and spectrum analyzers in that it was designed from its inception to be a computer-controlled system. The system was conceived and designed to provide maximum flexibility, and to minimize software and interface requirements. It utilizes current generation mini-computers, with a 16-bit word format. The system not only controls the RF sections, but also allows computation and analysis of the data obtained.

In order to build a receiving system which is completely computer compatible, the old concepts of swept-tuned local oscillators, tuned RF stages, and fixed IF frequencies and bandwidths had to be revised. The computer-controlled spectrum surveillance system described here begins with a synthesized local oscillator which, in effect, is a number of local oscillators synthesized from a